

TRIGONOMETRIC EQUATIONS

Section - 7

7.1 Some Basic Results

(i) If $\sin \theta = 0$, then $\theta = 0, \pm \pi, \pm 2\pi, \dots$

$$\Rightarrow \theta = n\pi \quad \text{where } n \in I.$$

(iii) If $\tan \theta = 0$ then, $\theta = 0, \pm \pi, \pm 2\pi, \dots$

$$\Rightarrow \theta = n\pi \quad \text{where } n \in I.$$

(v) If $\cos \theta = 1$ then, $\theta = 0, 2\pi, 4\pi, \dots$

$$\Rightarrow \theta = 2n\pi \quad \text{where } n \in I.$$

(ii) If $\cos \theta = 0$, then $\theta = \pm \pi/2, \pm 3\pi/2, \dots$

$$\Rightarrow \theta = (2n+1)\pi/2 \quad \text{where } n \in I.$$

(iv) If $\sin \theta = 1$ then, $\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$

$$\Rightarrow \theta = (4n+1)\frac{\pi}{2} \quad \text{where } n \in I.$$

(vi) If $\tan \theta = 1$ then, $\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \dots$

$$\Rightarrow \theta = (4n+1)\frac{\pi}{4} \quad \text{where } n \in I.$$

7.2 Some More Basic Results

(i) If $\sin \theta = \sin \alpha$
then $\theta = n\pi + (-1)^n \alpha$, where $n \in I$

(iii) If $\tan \theta = \tan \alpha$
then $\theta = n\pi + \alpha$, where $n \in I$

(v) If $\cos^2 \theta = \cos^2 \alpha$
then $\theta = n\pi \pm \alpha$, where $n \in I$

(ii) If $\cos \theta = \cos \alpha$
then $\theta = 2n\pi \pm \alpha$, where $n \in I$

(iv) If $\tan^2 \theta = \tan^2 \alpha$
then $\theta = n\pi \pm \alpha$, where $n \in I$

(vi) If $\sin^2 \theta = \sin^2 \alpha$
then $\theta = n\pi \pm \alpha$, where $n \in I$

Illustrating the Concepts :

(i) Find the values of θ satisfying $\sin \theta = \sin \alpha$.

(ii) Find the values of θ satisfying $\cos \theta = \cos \alpha$ in the interval $0 \leq \theta \leq \pi$.

(iii) Find the values of θ satisfying the equation : $\tan \theta = \tan \alpha$.

(i) $\sin \theta = \sin \alpha$

$$\Rightarrow \sin \theta - \sin \alpha = 0$$

$$\Rightarrow 2 \cos \frac{\theta + \alpha}{2} \sin \frac{\theta - \alpha}{2} = 0$$

$$\text{Either : } \cos \frac{\theta + \alpha}{2} = 0 \quad \text{or} \quad \sin \frac{\theta - \alpha}{2} = 0$$

$$\Rightarrow \frac{\theta + \alpha}{2} = (2l+1)\frac{\pi}{2} \quad \text{or} \quad \frac{\theta - \alpha}{2} = n\pi$$

[where l, n are integers]

$$\theta = (2l+1)\pi - \alpha \quad \text{or} \quad \theta = 2n\pi + \alpha$$

$$\theta = (\text{odd no.})\pi - \alpha \quad \text{or} \quad \theta = (\text{even no.})\pi + \alpha$$

$$\theta = n\pi + (-1)^n \alpha, \quad n \in I.$$

[where, $n \in I$]

$$(ii) \quad \cos \theta = \cos \alpha$$

$$\Rightarrow \cos \theta - \cos \alpha = 0$$

$$\Rightarrow -2 \sin \frac{\theta + \alpha}{2} \sin \frac{\theta - \alpha}{2} = 0$$

$$\text{Either } \sin \frac{\theta + \alpha}{2} = 0$$

$$\text{or } \sin \frac{\theta - \alpha}{2} = 0$$

$$\Rightarrow \frac{\theta + \alpha}{2} = n\pi$$

$$\text{or } \frac{\theta - \alpha}{2} = n\pi \quad [\text{where, } n \in I]$$

$$\Rightarrow \theta = 2n\pi - \alpha$$

$$\text{or } \theta = 2n\pi + \alpha \quad [\text{where, } n \in I]$$

Combining the two values, we get :

$$\theta = 2n\pi \pm \alpha \quad n \in I$$

$$(iii) \quad \tan \theta = \tan \alpha$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{\sin \alpha}{\cos \alpha}$$

$$\Rightarrow \sin \theta \cos \alpha - \cos \theta \sin \alpha = 0$$

$$\Rightarrow \sin (\theta - \alpha) = 0$$

$$\Rightarrow \theta - \alpha = n\pi, n \in I$$

$$\Rightarrow \theta = n\pi + \alpha, n \in I$$

$$(i) \quad \sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in I$$

$$(ii) \quad \cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$$

$$(iii) \quad \tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha, n \in I$$

Important : Every trigonometric equation should be manipulated so that it reduces to any of the above results.

Note : The following results should be committed to memory before proceeding further.

Illustration - 36 The number of value of x lying between 0 and 2π and satisfying the equation : $\sin x + \sin 3x = 0$ is are:

(A) 2

(B) 3

(C) 4

(D) 5

SOLUTION : (B)

The given equation is $\sin x + \sin 3x = 0$

$$2 \sin \frac{x + 3x}{2} \cos \frac{x - 3x}{2} = 0$$

$$\Rightarrow 2 \sin 2x \cos x = 0$$

$$\text{Either } \sin 2x = 0 \quad \text{or} \quad \cos x = 0$$

$$\Rightarrow 2x = n\pi, \quad \text{or} \quad x = (2n + 1) \pi/2, \quad [\text{where, } n \in I]$$

$$\Rightarrow x = n\pi/2, \quad \text{or} \quad x = (2n + 1) \pi/2, \quad [\text{where, } n \in I]$$

This is the general solution of the equation. To get particular solution satisfying $0 < x < 2\pi$, we will substitute integral values of n .

$$(i) \quad n = 0 \Rightarrow x = 0, \pi/2$$

$$(ii) \quad n = 1 \Rightarrow x = \pi/2, 3\pi/2$$

$$(iii) \quad n = 2 \Rightarrow x = \pi, 5\pi/2$$

$$(iv) \quad n = 3 \Rightarrow x = 3\pi/2, 7\pi/2$$

Hence for $0 < x < 2\pi$, the solution is $x = \pi/2, \pi, 3\pi/2$.